AN APPROXIMATE SOLUTION OF THE PROBLEM OF A STATIONARY INDUCED HIGH FREQUENCY DISCHARGE IN AN ENCLOSED VOLUME

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A steady-state high frequency discharge excited in an enclosed volume where there is no gas flow has a series of characteristic properties. Firstly, there appear different mechanisms governing the formation of the discharge depending on the pressure of the working

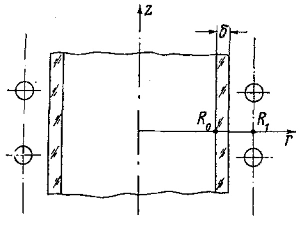

Fig. 1

gas. Thus at low pressures the predominant mechanism is the diffusion of particles to the walls of the flask as covered in [1]. At high pressures a mechanism connected with heat conduction comes into operation [2]. Secondly, in such a discharge energy transfer from the external field to the plasma does not occur over the whole volume occupied by the discharge, as a result of which the discharge geometry is determined not only by the method of introducing energy into the discharge, but also by the nature of its distribution and the method by which it is extracted from the discharge. Depending on these factors either a ring discharge or a cylindrical discharge is formed.

A high-frequency discharge by virtue of the very conditions of its formation, is unstable. However, in the case of a dense plasma the departure of the temperature components from their values for equilibrium temperature is as a rule not large. We shall consider a dense plasma in which the governing mechanism is the heat conduction. We can be fairly accurate in assuming that there exists local thermodynamic equilibrium in such a plasma. Then all transport coefficients are functions of temperature and pressure, and our problem reduces to solving the energy balance equation together with Maxwell's equations.

While the present paper was being prepared for the press there appeared an article [3] in which a similar system of equations was treated dealing with a high-frequency high-pressure induced discharge. In this paper instead of solving the appropriate boundary value problem the authors solve the Cauchy problem numerically, arbitrarily specifying supplementary conditions on the axis of the discharge. In our opinion such a procedure has three serious drawbacks: first, there is no possibility of examining qualitatively the functional connections of the plasma parameters; secondly, the arbitrary choice of the supplementary conditions makes it hard to compare calculated and experimental results; thirdly, it is impossible to investigate an important series of questions connected with discharge contraction.

The present article solves the boundary value problem by the method of successive approximations. The first approximation to the solution enables us to make a qualitative examination of phenomena taking place in an induced discharge. The appropriate results which are useful for practical application will be set out in another article. The second approximation to the solution is used for numerical calculations of the basic discharge parameters in argon and comparing them with experiment. A. P. Sobolev is responsible for the theoretical portion of this article.

1. Statement of the problem and derivation of the fundamental system of equations. We shall consider a system composed of a cylindrical inductor having a gas filled container placed within it (Fig. 1). We shall assume that 1) the field exciting the discharge is fairly uniform along the axis of the discharge; 2) the plasma is as a whole stationary; 3) the pressure in the system is constant; 4) the wavelength of the electromagnetic field is much larger than the characteristic dimension of the plasma; 5) the radiated energy may be neglected in comparison with the heat losses of the plasma.

Employing the methods of nonequilibrium thermodynamics and assuming the assumptions to be fulfilled, we now write out the system of equations for an induced high frequency discharge in the form

$$-\frac{\partial H}{\partial r} = \frac{4\pi}{c}\sigma E, \quad \frac{1}{r}\frac{\partial}{\partial r} rE = -\frac{1}{c}\frac{\partial H}{\partial t},$$
$$\rho\frac{\partial u}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r\lambda\frac{\partial T}{\partial r} + \sigma E^2. \tag{1.1}$$

Here H, E are the magnetic and electric field strengths, λ is the coefficient of thermal conductivity, σ is the coefficient of electrical conductivity, T is the temperature of the plasma, ρ is the plasma density, and U is the internal energy per unit volume of the plasma.

Since the plasma is heated by a high frequency field all quantities are functions of two characteristic times; t_0—the time for establishing a steady state and $t_1 = 1/\omega$, where ω is the frequency of the electromagnetic field. When the last equation of (1.1) is averaged with respect to its rapidly varying argument $\tau = \omega t$ the system of equations (1.1) for the steady state may be written in the form

$$-\frac{\partial H}{\partial r} = \frac{4\pi}{c}\sigma E, \quad \frac{1}{r}\frac{\partial}{\partial r} rE = -\frac{\omega}{c}\frac{\partial H}{\partial \tau},$$
$$\frac{1}{r}\frac{\partial}{\partial r} r\lambda\frac{\partial T}{\partial r} + \frac{\sigma}{2\pi}\int_0^{2\pi} E^2(\tau)\,d\tau = 0. \tag{1.2}$$

The boundary conditions for this system of equations are

$$H(R_0) = \frac{4\pi}{c} nJ_0,$$
$$\lambda\frac{\partial T}{\partial r}\bigg|_{r=R_0} = -\alpha_1[T(R_0) - T_0]; \tag{1.3}$$

$$\alpha_1 = \alpha\frac{1 + \delta/R_0}{1 + \alpha(R_0 + \delta)\langle\lambda\rangle^{-1}\ln(1 + \delta/R_0)}. \tag{1.4}$$

Here nJ is the number of inductor ampere turns per unit length, T_0 is the temperature of the surrounding medium, α is the heat transfer coefficient, $\langle\lambda\rangle$ is the mean thermal conductivity coefficient of the wall in the range of temperatures from $T(R_0)$ to $T(R_0 + \delta)$.

If the current in the inductor varies as $J = J_0 \cos\tau$, then a solution of the system of equations (1.2) may be sought in the form

$$E = E_1\cos\tau + E_2\sin\tau, \quad H = H_1\cos\tau + H_2\sin\tau. \tag{1.5}$$

Introducing the dimensionless coordinate $x = (r/R_0)^2$ (1.6) and the dimensionless functions proportional to the squared amplitudes of the electric and magnetic fields

$$\varphi = (E_1^2 + E_2^2)\left(\frac{c^2}{2\pi\omega R_0 nJ_0}\right)^2, \qquad \psi = (H_1^2 + H_2^2)\left(\frac{c}{4\pi nJ_0}\right)^2, \tag{1.7}$$

and using expression (1.5) we can write the system of equations in the form

$$\frac{d^2 x\varphi}{dx^2} = 2\psi, \qquad \frac{c^2}{\pi\omega R_0^2\sigma(T)}\frac{d}{dx}\frac{c^2}{\pi\omega R_0^2\sigma(T)} x\frac{d\psi}{dx} = 2\varphi,$$
$$\frac{d}{dx} x\lambda(T)\frac{dT}{dx} + \frac{1}{2}\left(\frac{\pi\omega R_0^2 nJ_0}{c^2}\right)^2\sigma(T)\varphi = 0. \tag{1.8}$$

The boundary conditions are

$$\varphi(0)=0,\quad \psi(1)=1;\quad \frac{d\varphi}{dx}\Big|_{x=0},\quad \frac{d\psi}{dx}\Big|_{x=0}<M=\text{const},$$

$$\frac{dT}{dx}\Big|_{x=0}=0,\quad \lambda\frac{dT}{dx}\Big|_{x=1}=-\frac{\alpha_1 R_0}{2}[T(1)-T_0]. \qquad (1.9)$$

With the help of the first two equations of (1.8) the last equation of the system may be written as follows:

$$F(T)=\int\limits_{T(1)}^{T(x)}\lambda(T)\sigma(T)\,dT=\left(\frac{nJ_0}{2}\right)^2[1-\psi(x)], \qquad (1.10)$$

and system (1.8) together with the boundary conditions (1.9) may be rewritten in the form

$$\varphi(x)=2\int\limits_0^x\left(1-\frac{\xi}{x}\right)\psi(\xi)\,d\xi,$$

$$\psi(x)=1-2\int\limits_x^1\frac{\pi\omega R_0^2}{c^2}\sigma(T)\frac{d\xi}{\xi}\int\limits_0^\xi\varphi(\tau)\frac{\pi\omega R_0^2}{c^2}\sigma(T)\,d\tau,$$

$$F(T)=\int\limits_{T(1)}^{T(x)}\lambda(T)\sigma(T)\,dT=\left(\frac{nJ_0}{2}\right)^2[1-\psi(x)],$$

$$2[T(1)-T_0]\,\alpha_1R_0\sigma[T(1)]=(nJ_0)^2\frac{d\psi}{dx}\Big|_{x=1}. \qquad (1.11)$$

For $\sigma[T(1)] = 0$ system (1.11) has a trivial solution, and the field is the same as the field in the absence of the plasma. This means that there does not exist an equilibrium discharge with a conductivity which is zero at the wall. We shall therefore assume that there exists a boundary temperature $T = T^*$ such that $\sigma(T^*) > 0$ and $\sigma(T) = 0$ for $T < T^*$, $T^* > T(1)$.

On this assumption the system of equations (1.11) assumes the form

$$\varphi(x)=2\int\limits_0^x\left(1-\frac{\xi}{x}\right)\psi(\xi)\,d\xi,$$

$$\psi(x)=1-2\int\limits_x^{x^*}\frac{\pi\omega R_0^2}{c^2}\sigma(T)\frac{d\xi}{\xi}\int\limits_0^\xi\frac{\pi\omega R_0^2}{c^2}\sigma(T)\varphi(\tau)\,d\tau,$$

$$F(T)=\int\limits_{T^*}^{T(x)}\lambda(T)\sigma(T)\,dT=\left(\frac{nJ_0}{2}\right)^2[1-\psi(x)],$$

$$\int\limits_{T(1)}^{T^*}\lambda(T)\,dT=\frac{\alpha_1R_0}{2}[T(1)-T_0]\ln\frac{1}{x^*},$$

$$\frac{4\pi\omega R_0^2}{c^2}\left(\frac{nJ_0}{2}\right)^2\int\limits_0^{x^*}\frac{\pi\omega R_0^2}{c^2}\sigma(T)\varphi(x)\,dx=\alpha_1R_0[T(1)-T_0]. \qquad (1.12)$$

2. Solution of the one dimensional problem. We shall look for a solution of the system of equations obtained above by the method of successive approximations. For the first approximation we shall set

$$T^{(0)}(x)=\text{const},\quad x^{*(0)}=1$$

and determine the subsequent approximations by the system of equations

$$\varphi^{(n)}(x)=2\int\limits_0^x\left(1-\frac{\xi}{x}\right)\psi^{(n)}(\xi)\,d\xi,$$

$$\psi^{(n)}(x)=1-2\int\limits_x^{x^{*(n-1)}}\frac{\pi\omega R_0^2}{c^2}\sigma[T^{(n-1)}]\frac{d\xi}{\xi}\times$$

$$\times\int\limits_0^\xi\varphi^{(n)}(\tau)\frac{\pi\omega R_0^2}{c_2}\sigma[T^{(n-1)}]\,d\tau,$$

$$F(T^{(n)})=\int\limits_{T^*}^{T^{(n)}(x)}\lambda(T)\sigma(T)\,dT=\left(\frac{nJ_0}{2}\right)^2[1-\psi^{(n)}(x)],$$

$$\int\limits_{T^{(n)}(1)}^{T^*}\lambda(T)\,dT=\frac{\alpha_1R_0}{2}[T^{(n)}(1)-T_0]\ln\frac{1}{x^{*(n)}},$$

$$\frac{4\pi\omega R_0^2}{c^2}\left(\frac{nJ_0}{2}\right)^2\int\limits_0^{x^{*(n-1)}}\frac{\pi\omega R_0^2}{c^2}\sigma[T^{(n-1)}]\varphi^{(n)}(x)\,dx=$$

$$=\alpha_1R_0[T^{(n)}(1)-T_0]. \qquad (2.1)$$

We shall show that the best first approximation, in the sense that it has the least upper uniform bound, is obtained for the functions $T^{(n)}_{(x)}$ by solving the equations

$$F(T)=\left(\frac{nJ_0}{2}\right)^2\left(1-\frac{1}{(\operatorname{ber}\sqrt{\beta})^2+(\operatorname{bei}\sqrt{\beta})^2}\right),$$

$$\beta=\frac{4\pi\omega R_0^2\sigma(T)}{c^2}. \qquad (2.2)$$

We shall take the preliminary step of proving that if $T_1(x) \geq T_2(x)$ for $0 \leq x \leq 1$, and $\psi_1(x)$, $\psi_2(x)$ are solutions of the system of equations

$$\varphi(x)=2\int\limits_0^x\left(1-\frac{\xi}{x}\right)\psi(\xi)\,d\xi, \qquad (2.3)$$

$$\psi(x)=1-2\int\limits_x^1\frac{\pi\omega R_0^2}{c^2}\sigma(T)\frac{d\xi}{\xi}\int\limits_0^\xi\varphi(\tau)\frac{\pi\omega R_0^2}{c^2}\sigma(T)\,d\tau \qquad (2.4)$$

for $T = T_1(x)$ and $T = T_2(x)$, respectively, then $\psi_1(0) \leq \psi_2(0)$. We shall assume that under the conditions laid down $\psi_1(x) > \psi_2(x)$ for all $x \in [0, 1]$. Then it follows from Eq. (2.3) that

$$\varphi_1(x)-\varphi_2(x)=2\int\limits_0^x\left(1-\frac{\xi}{x}\right)[\psi_1(\xi)-\psi_2(\xi)]\,d\xi>0;$$

but we then have from Eq. (2.4) for $x = 0$

$$\psi_1(0)-\psi_2(0)=-2\int\limits_0^1\left(\frac{\pi\omega R_0^2}{c_2}\right)^2\Big\{\sigma(T_1)\int\limits_0^\xi\varphi_1(\tau)\sigma(T_1)\,d\tau-$$

$$-\sigma(T_2)\int\limits_0^\xi\varphi_2(\tau)\sigma(T_2)\,d\tau\Big\}\frac{d\xi}{\xi}<0$$

$$(\sigma(T_1)>\sigma(T_2)\quad\text{for}\quad T_1>T_2). \qquad (2.5)$$

Since the functions $\psi(x)$ are continuous it follows that $\psi_1(x) \leq \psi_2(x)$ somewhere in the neighborhood of the point $x = 0$. Thus we arrive at a contradiction. This means that either $\psi_1(x) < \psi_2(x)$ for all $x \in [0, 1)$, or else there exists at least one point $x = x_n$ such that $\psi_1(x_n) = \psi_2(x_n)$ for $0 \leq x_n < 1$; then if x_1 is the first such point and $\psi_1(x) > \psi_2(x)$ for $0 \leq x < x_1$, then $\varphi_1(x) > \varphi_2(x)$ and

$$\psi_1(0)-\psi_2(0)=-2\int\limits_0^x\left(\frac{\pi\omega R_0^2}{c^2}\right)^2\Big\{\sigma(T_1)\int\limits_0^\xi\varphi_1(\tau)\sigma(T_1)\,d\tau-$$

$$-\sigma(T_2)\int\limits_0^\xi\varphi_2(\tau)\sigma(T_2)\,d\tau\Big\}\frac{d\xi}{\xi}<0,$$

i.e., we again arrive at a contradiction. Consequently $\psi_1(0) \leq \psi_2(0)$.

We shall now assign the arbitrary value T. Some solution $\psi_0(x)$ of

$$F(T_1)<(^1/_2nJ_0)^2.$$

We now determine T by the equation $F(T) = (nJ_0/2)^2$, then $T_1(0) < T$, and consequently $T_1(x) < T$. Using the proof given above for the functions $T_1(x) = T$ and $T_2(x) = T_1(0)$ we obtain $\psi_1(0) > \psi_2(0)$. If $T_2(0)$ is determined by the relation $F[T_2(0)] = (nJ_0/2)^2(1 - \psi_1(0))$, then $T_2(0) < T_1(0) < T$.

If the process is continued, we obtain an infinite decreasing series of numbers $T_n(0)$, bounded above by the value of T, and below by T*.

Table 1

$T°$ K	$\lambda_0 \cdot 10^{-5}$ erg/cm sec g	W, kW/cm	$\frac{r^*}{R_0}$
370	1.44	0.12	0.854
470	1.52	0.30	0.941
570	1.59	0.52	0.963
700	1.68	0.81	0.977
800	1.73	1.05	0.982
900	1.78	1.29	0.981
1000	1.82	1.54	0.988
1100	1.86	1.80	0.991
1200	1.90	2.06	0.992

Consequently there exists a limit $T^{(0)}$ of this sequence. The solution of the system of equations (2.3) and (2.4) for $T(x) = T_n(0) = \text{const}$ has the form

$$\varphi_n = \frac{4}{\beta_n} \frac{(\mathrm{bei}' \sqrt{\beta_n x})^2 + (\mathrm{ber}' \sqrt{\beta_n x})^2}{(\mathrm{ber} \sqrt{\beta_n})^2 + (\mathrm{bei} \sqrt{\beta_n})^2},$$

$$\psi_n = \frac{(\mathrm{ber} \sqrt{\beta_n x})^2 + (\mathrm{bei} \sqrt{\beta_n x})^2}{(\mathrm{ber} \sqrt{\beta_n})^2 + (\mathrm{bei} \sqrt{\beta_n})^2} \quad \left(\beta_n = \frac{4\pi\omega R_0^2 \sigma(T_n)}{c^2}\right). \tag{2.6}$$

If we go to the limit as $n \to \infty$ in the relation $F(T_{n+1}) = (nJ_0/2)^2(1 - \psi_n(0))$, we obtain the required relation (2.2). In order to find subsequent approximations it is necessary to solve a system of integral equations composed of the first two equations of system (2.1). It may be reduced to the form

$$\varphi^{(n)}(x) = 1 + \int_0^x \varphi^{(n)}(\nu) K(x, \nu)\, d\nu,$$

$$\varphi^{(n)}(x) = 2\int_0^x \left(1 - \frac{\xi}{x}\right) \psi^{(n)}(\xi)\, d\xi,$$

$$\psi^{(n)}(x) = \frac{\varphi^{(n)}(x)}{\varphi^{(n)}(x^{*(n-1)})},$$

$$K(x, \nu) = 2\int_\nu^x \left(\int_\tau^x \frac{\pi\omega R_0^2}{c^2} \sigma[T^{(n-1)}] \frac{d\xi}{\xi}\right)^2 d\tau. \tag{2.7}$$

Numerical calculations have shown that with an accuracy better than 15% the solution of the system of equations (2.7) may be replaced by the approximate expression

$$\psi^{(n)}(x) = \frac{(\mathrm{ber} \sqrt{\beta_n x})^2 + (\mathrm{bei} \sqrt{\beta_n x})^2}{(\mathrm{ber} \sqrt{\beta_n x^{*(n-1)}})^2 + (\mathrm{bei} \sqrt{\beta_n x^{*(n-1)}})^2},$$

$$\varphi^{(n)}(x) = \frac{4}{\beta_n} \frac{(\mathrm{bei}' \sqrt{\beta_n x})^2 + (\mathrm{ber}' \sqrt{\beta_n x})^2}{(\mathrm{ber} \sqrt{\beta_n x^{*(n-1)}})^2 + (\mathrm{bei} \sqrt{\beta_n x^{*(n-1)}})^2}. \tag{2.8}$$

In practice it turns out that in order to obtain solutions which are just as accurate as the initial data concerning the thermal conductivity and electrical conductivity, two approximations are enough.

The first approximation to the solution has the form

$$\int_{T^*}^{T^{(1)}(x)} \lambda(T)\sigma(T)\, dT = \left(\frac{nJ_0}{2}\right)^2 \times$$

$$\times \left(1 - \frac{(\mathrm{ber} \sqrt{\beta_1 x})^2 + (\mathrm{bei} \sqrt{\beta_1 x})^2}{(\mathrm{ber} \sqrt{\beta_1})^2 + (\mathrm{bei} \sqrt{\beta_1})^2}\right),$$

$$\int_{T^{(1)}(1)}^{T^*} \lambda(T)\, dT = \frac{\alpha_1 R_0}{2} [T^{(1)}(1) - T_0] \ln \frac{1}{x^{*(1)}},$$

$$\beta_1 \left(\frac{nJ_0}{2}\right)^2 \int_0^1 \frac{\sigma[T^{(1)}(x)]}{\sigma^2[T^{(1)}(0)]} \frac{(\mathrm{ber}' \sqrt{\beta_1 x})^2 + (\mathrm{bei}' \sqrt{\beta_1 x})^2}{(\mathrm{bei} \sqrt{\beta_1})^2 + (\mathrm{ber} \sqrt{\beta_1})^2}\, dx =$$

$$= \alpha_1 R_0 [T^{(1)}(1) - T_0]. \tag{2.9}$$

The second approximation is

$$\int_{T^*}^{T^{(2)}(x)} \lambda(T)\sigma(T)\, dT = \left(\frac{nJ_0}{2}\right)^2 \times$$

$$\times \left(1 - \frac{(\mathrm{ber} \sqrt{\beta_2 x})^2 + (\mathrm{bei} \sqrt{\beta_2 x})^2}{(\mathrm{ber} \sqrt{\beta_2 x^{*(1)}})^2 + (\mathrm{bei} \sqrt{\beta_2 x^{*(1)}})^2}\right),$$

$$\int_{T^{(2)}(1)}^{T^*} \lambda(T)\, dT = \frac{\alpha_1 R_0}{2} [T^{(2)}(1) - T_0] \ln \frac{1}{x^{*(2)}},$$

$$\beta \left(\frac{nJ_0}{2}\right)^2 \int_0^{x^{*(1)}} \frac{\sigma[T^{(2)}(x)]}{\sigma^2[T^{(2)}(0)]} \times$$

$$\times \frac{(\mathrm{bei}' \sqrt{\beta_2 x})^2 + (\mathrm{ber}' \sqrt{\beta_2 x})^2}{(\mathrm{ber} \sqrt{\beta_2 x^{*(1)}})^2 + (\mathrm{bei} \sqrt{\beta_2 x^{*(1)}})^2}\, dx =$$

$$= \alpha_1 R_0 [T^{(2)}(1) - T_0]. \tag{2.10}$$

This system of equations was solved numerically.

3. Calculation of the Discharge Parameters and Comparison with Experiment. Basic data for the calculations were taken from experimental papers [2,4], namely an argon discharge at a pressure of 1 atm, induced in a closed flask (with no gas flow) of radius $R_0 = 3.75$ cm with quartz walls of thickness $\delta = 0.2$ cm. The frequency of the generator was 12 Mc/sec. The experimental papers referred to also investigated a xenon discharge, but calculations cannot be made for this at present because of the lack of complete and reliable data on the temperature dependence of the thermal conductivity coefficient of xenon.

The following integrals must be calculated in order to carry out the calculations:

$$J_1 = \int_T^{T^*} \lambda_1(T)\, dT \quad \text{for} \quad T \leqslant 1.5 \cdot 10^3\ °\mathrm{K}, \tag{3.1}$$

$$J_2 = \int_{T^*}^{T} \lambda_1(T)\sigma(T)\, dT \quad \text{for} \quad T > T^*, \tag{3.2}$$

where the value $T^* = 4500$°K was chosen as the boundary temperature.

Table 2

$T(0) \cdot 10^{-3}$° K	$n^*J_0\ \frac{\text{A} \cdot \text{turn}}{\text{cm}}$	W $\frac{\text{kW}}{\text{cm}}$	$\frac{r^*}{R_0}$	$\frac{r_1}{R_0}$
8.0	13.3	0.21	0.907	0.820
8.5	17.7	0.36	0.948	0.870
9.0	22.6	0.54	0.964	0.906
10.0	33.0	1.06	0.983	0.940
10.5	39.2	1.43	0.987	0.945

Estimates indicate that for a small degree of ionization the thermal conductivity associated with diffusion processes may be neglected in comparison with normal thermal conductivity. Thus in what follows, instead of the total thermal conductivity coefficient $\lambda_1(T)$, we shall use the coefficient of thermal conductivity $\lambda(T)$ due to conduction. Data on its values for temperature up to 1500° K are contained in papers [5, 6], and for temperatures from $2 \cdot 10^3$ to $30 \cdot 10^3$° K are contained in paper [7].

The integrals J_1 and J_2 were calculated numerically using Simpson's rule with a step $\Delta T = 100°$ for J_1 and $\Delta T = 500°$ for J_2. The electrical conductivity was calculated in accordance with paper [8].

For subsequent comparison of calculations with experiment the relation between the number of amp turns nJ_0, appearing in the initial equations, and the value of the power extracted from the discharge by heat flow must be established. The power leaving the discharge per unit length is

$$W = -2\pi R_0 \lambda \left.\frac{\partial T}{\partial r}\right|_{r=R_0} = 2\pi R_0 \alpha_1 [T(1) - T_0]. \qquad (3.3)$$

The required relation between the power and the ampere turns may be obtained from the third of Eqs. (2.10) taking (3.3) into account:

$$W = \frac{2\pi\beta}{\sigma[T^{(2)}(0)]}\left(\frac{nJ_0}{2}\right)^2 \int_0^{x^*(1)} \frac{\sigma[T^{(2)}(x)]}{\sigma[T^{(2)}(0)]} \times$$

$$\times \frac{(\mathrm{ber}'\sqrt{\beta x})^2 + (\mathrm{bei}'\sqrt{\beta x})^2}{(\mathrm{ber}\sqrt{\beta x^{*(1)}})^2 + (\mathrm{bei}\sqrt{\beta x^{*(1)}})^2}\, dx. \qquad (3.4)$$

Using the second of Eqs. (2.10) we may establish a relation between the power leaving unit area of the wall and the bounding

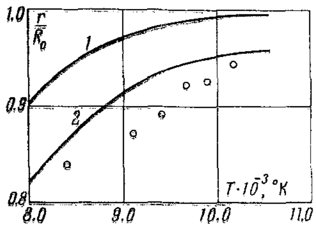

Fig. 2

coordinate x^* (or the relative bounding radius $r^*/R_0 = (x^*)^{1/2}$) with the temperature of the inner surface of the flask T(1). The results of the appropriate calculation, assuming that there exists an adjusted heat exchange, are given in Table 1. The heat exchange is taken to be adjusted if the inequality $\alpha\delta/\lambda_0 \gg 1$ is fulfilled, where α is the coefficient of heat transfer from the wall to the cooling medium, λ_0 is the mean value of the thermal conductivity coefficient of the wall material in the temperature range between the external and internal surfaces.

The results given in Table 1 enable us to evaluate the limiting values of power removed from unit area of the wall under ideal cooling conditions. The criterion of the maximum allowable load will be taken to be the surface temperature of the inner wall at which there is a danger of its softening. For quartz glass this is roughly 1200°K, for which $W \approx 90$ W/cm².

The basic results obtained from solving the systems of equations (2.9) and (2.10) for the case in question are given in Table 2. The first column gives the values of maximum temperature in the discharge, the second gives the number of amp turns per cm length of the inductor, the third gives the power of the heat flow leaving unit surface, while the fourth gives the relative diameter of the electrically conducting region with a bounding temperature of 4500°K.

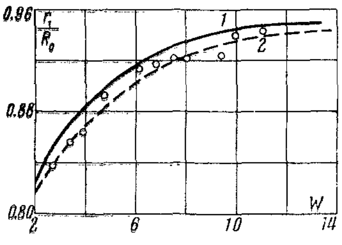

Fig. 3

In order to compare theoretical and experimental results for the boundary of the discharge we must establish precisely the conditions required for such a comparison to be correct. Actually in carrying out the theoretical calculations it was assumed that the discharge occupies a region having appreciable electrical conductivity. The isotherm corresponding to 4500°K was chosen as the conditional boundary of the discharge. In paper [2] the region of emission was studied by photographic methods, and not by using the electrical conductivity of the discharge, while its boundary was taken to be a circle whose radius corresponded to an assumed drop in radiation

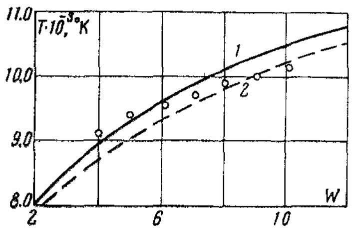

Fig. 4

intensity to one tenth of the maximum value. Subsequently after subtracting the characteristic curve of the film used, it was established that drop in intensity to one tenth as determined from the darkening curve of the negative, corresponds to an actual drop of radiation intensity roughly equal to one half. Thus the discharge boundary in the experiment was established using the condition I = 0.5 I(0). In order to compare experimental and calculated results for every possible value of the maximum temperature, the relative radius of the illuminated zone boundary r_1 was calculated corresponding to the condition given above. Results of the calculations are to be found in the fifth column of Table 2.

We must also allow for the fact that the one-dimensional case treated by the theory was compared with an experiment performed under conditions when the ratio of the discharge diameter to the height of the inductor was of the order of unity, i.e., when the processes of heat removal and energy transfer from the generator to the plasma are plainly of a two dimensional character. A correction for the two-dimensional nature of the real problem in the process of energy transfer from the generator to the plasma is made by introducing an effective number of inductor turns n^* into the solution of the one dimensional problem. Its magnitude is determined so that $H = 4\pi n^* J_0/c$ coincides with the magnetic field strength created by an inductor in the absence of a plasma at the point $r = R_0$, $z = 0$.

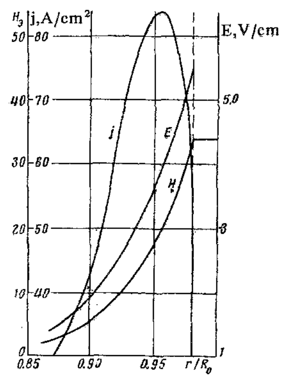

Fig. 5

The two dimensional nature of the heat removal may be allowed for by introducing an effective length of the zone l^*, through which the heat is removed. Considerable computational difficulties are involved in determining l^* theoretically, and in order to avoid them this quantity was estimated from experimental data. Taking the results of papers [2,4] and eliminating the "discharge power" parameter we obtain an expression for the radius of the luminous zone as a function of the maximum discharge temperature. The same function was calculated. The comparison of experimental and calculated data given in Fig. 2 shows that they agree quite satisfactorily. Here 1 was calculated for a boundary at T = 4500°K, while 2 was calculated for the boundary of the luminous zone, and experimental data are denoted by points. It follows from this that at the same temperatures the experimental and calculated values of the power are in agreement with each other. Assuming that for a given maximum temperature the calculated and experimental values of the total energy introduced into the plasma are equal, we may determine the effective dimension from which this energy is extracted. In so doing we must subtract from the experimental value for the power the radiative losses estimated for argon under a pressure of 1 atm at 15–17% of the total power [9]. In the case under consideration $l^* = 9.5$ cm.

Using the value of l^* thus obtained the relative plasma radius and maximum temperature were determined as functions of the discharge power W. Comparisons of calculated and experimental results are given in Fig. 3 and Fig. 4, respectively. Curves 1 in both figures were constructed without allowing for radiation, while curves 2 allowed for it. Experimental data are shown by points. The calculations and experiment are in quite satisfactory quantitative agreement. Some qualitative discrepancy in the behavior of the experimental and calculated functions in Fig. 4 is connected in the regions of low power with an appreciable decrease in the radius of the discharge, which leads to l^* becoming nonconstant. As the power increases, the fact that approximate equations were used instead of the original integral expressions begins to assert itself. As was pointed out above the approximation does not fall short of the exact solution by more than 15%.

An estimate of the electric and magnetic field strengths and their radial distributions, as well as the current-density distribution in the discharge for the case when $T(0) = 9.5 \cdot 10^3$ °K are given in Fig. 5. Here the vertical dotted line is the boundary of the conducting zone. The depth of the skin layer was estimated from the current density curve, and was roughly 4 mm in the case in question.

In order to make an approximate estimate of the electric field strength at the plasma boundary in the case when $\beta \gg 1$, we obtain the following relation from the theory:

$$E(x^*) = \frac{nJ_0}{R_0 \sigma [T(0)]} \sqrt{\beta}. \qquad (3.6)$$

In the case when we actually measure the values nJ_0 and $\beta \gg 1$, we may make a direct estimate of the maximum discharge temperature. We give the values of the maximum discharge temperature calculated for some values of inductor amp turns:

$$nJ_0 = 15, \quad 20, \quad 25, \quad 30, \quad 35, \quad 40 \text{ A·turn/cm}$$

$$T(0) \cdot 10^{-3} = 8.2, \quad 8.8, \quad 9.3, \quad 9.8, \quad 10.2 \quad 10.5\text{° K}.$$

If the height of the inductor is commensurable with the discharge diameter, then we must use the effective value for the number of inductor turns n^*.

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REFERENCES

1. H. U. Eckert, Diffusion Theory of the Electrodless Ring Discharge, J. Appl. Phys. vol. 33, no. 9, 1962.

2. R. E. Rovinskii, L. E. Belousova, and V. A. Gruzdev, "The geometry of an electrodeless discharge induced in inert gases," Teplofiz. vysokikh temperatur, vol. 4, no. 3, 1966.

3. V. N. Soshnikov and E. S. Trekhov, "On the theory of high-frequency, high-pressure vortex discharges, Part I," Teplofiz. vysokikh temperatur, vol. 4, no. 2, 1966.

4. R. E. Rovinskii, V. A. Gruzdev, T. M. Gutenmakher, and A. P. Sobolev, "Temperature determination in a steady state high frequency induced discharge," Teplofiz. vysokikh temperatur, no. 2.

5. P. Z. Nuttal, "Tables of heat properties of gases," US Depart. Nat. Bur. Standards. Washington D. C., 1955.

6. J. Hilsenrath, "Tables of heat properties of gases," US Depart. Nat. Bur. Standards. Washington D. C., 1951.

7. P. P. Kulik, I. G. Panevin, and V. I. Khvasyuk, "Theoretical calculation of viscosity, thermal conductivity, and the Prandtl criterion of argon in the presence of ionization," Teplofiz. vysokikh temperatur, vol. 1, no. 1, 1963.

8. A. Sherman, "Calculation of electrical conductivity of ionized gases," ARS—Journal, vol. 30, no. 6. 1960.

9. R. E. Rovinskii, V. A. Gruzdev, and I. I. Shirokova, "The energy balance of a steady state induced discharge," Teplofiz. vysokikh temperatur, vol. 4, no. 1, 1966.

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